GCE AS/A LEVEL

2305U10-1

Z22-2305U10-1

MONDAY, 16 MAY 2022 – AFTERNOON

FURTHER MATHEMATICS – AS unit 1
FURTHER PURE MATHEMATICS A

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. The complex numbers z, w are given by $z = 3 - 4\mathrm{i}$, $w = 2 - \mathrm{i}$.

(a) (i) Find the modulus and argument of zw.

(ii) Express zw in the form $r(\cos\theta + \mathrm{i}\sin\theta)$. [5]

(b) The complex numbers v, w, z satisfy the equation $\dfrac{1}{v} = \dfrac{1}{w} - \dfrac{1}{z}$.

Find v in the form $a + \mathrm{i}b$, where a, b are real. [5]

(c) The complex conjugate of v is denoted by $\bar{v}$.

Show that $v\bar{v} = k$, where k is a real number whose value is to be determined. [2]

2. (a) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{pmatrix} 3 & 4 \\ -1 & -2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -11 \\ 7 \end{pmatrix}.$$

Given that $\mathbf{AX} = \mathbf{B}$, find the matrix $\mathbf{X}$. [4]

(b) (i) Find the 2 × 2 matrix, $\mathbf{T}$, which represents a reflection in the line $y = -2x$.

(ii) The images of the points $C(2, 7)$ and $D(3, 1)$, under $\mathbf{T}$, are E and F respectively.

Find the coordinates of the midpoint of EF. [7]

3. The vector equation of the line *L* is given by

$$\mathbf{r} = -\mathbf{i} + 2\mathbf{j} - 6\mathbf{k} + \lambda(4\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}).$$

The Cartesian equation of the plane Π is given by

$$3x + 8y - 9z = 0.$$

Find the Cartesian coordinates of the point of intersection of *L* and Π. [5]

4. The positive integer N is such that $1^2 + 2^2 + 3^2 + \ldots + N^2 = (3N - 2)^2$.

Write down and simplify an equation satisfied by N. Hence find the possible values of N. [7]

5. (a) The complex number z is represented by the point $P(x, y)$ in an Argand diagram. Given that

$$|z - 3 + 2\mathrm{i}| = |z - 3|,$$

find the equation of the locus of *P*. [3]

(b) Give a geometric interpretation of the locus of *P*. [1]

6. The roots of the cubic equation

$$2x^3 + px^2 - 126x + q = 0$$

form a geometric progression with common ratio −3.

Find the possible values of p and q, giving your answers in surd form. [8]

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7. The vector equations of the lines L_1, L_2, L_3 are given by

$$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + n\mathbf{j} + \mathbf{k}),$$

$$\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 3\mathbf{k}),$$

$$\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + v(p\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}),$$

respectively, where n and p are constants.

The line L_1 is perpendicular to the line L_2. The line L_1 is also perpendicular to the line L_3.

(a) Show that the value of n is –3 and find the value of p. [3]

(b) Find the acute angle between the lines L_2 and L_3. [4]

8. The point (x, y, z) is rotated through 60° anticlockwise around the z-axis.
After rotation, the value of the x-coordinate is equal to the value of the y-coordinate.

Show that $y = \left(a + \sqrt{b}\right)x$, where a, b are integers whose values are to be determined. [7]

9. (a) Given that $A_r = \dfrac{1}{r+1} - \dfrac{2}{r+2} + \dfrac{1}{r+3}$, show that A_r can be expressed as $\dfrac{2}{(r+1)(r+2)(r+3)}$. [2]

(b) Hence, show that $\displaystyle\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)} = \frac{1}{6} - \frac{1}{(n+2)(n+3)}$. [5]

(c) Find the ratio of $\displaystyle\sum_{r=1}^{5} A_r : \sum_{r=1}^{10} A_r$, giving your answer in its simplest form. [2]

END OF PAPER